

## Natural Deduction Exercises with Derived Rules

PHI 154 (Eliot) Fall 2023, version 2023-11-08

On average (but not necessarily), the second half of these is more difficult than the first half. I have tried to include problems that encourage you to practice the derived rules. It will benefit you to do as many as you can until you feel that proving them feels fairly quick and natural.

The following are valid arguments in TFL or are theorems that can be proved without premises. Prove them using the natural deduction system for TFL including the derived rules.

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| <p>1. <math>\frac{\neg N \quad (\neg N \rightarrow L) \wedge [D \leftrightarrow (\neg N \vee A)]}{L \wedge D}</math></p>                     | <p>14. <math>\frac{\neg\neg O \wedge \neg\neg S \quad \frac{\neg O \vee \neg T}{\neg T}}{\neg T}</math></p>   |
| <p>2. Derive <math>(A \wedge A) \leftrightarrow A</math> without premises.</p>   | <p>15. <math>\frac{\neg R \rightarrow P \quad (F \vee R) \wedge (O \wedge M) \quad \frac{O \rightarrow \neg R}{M \wedge P}}{M \wedge P}</math></p>  |
| <p>3. <math>\frac{M \rightarrow (A \rightarrow R) \quad \neg A \rightarrow \neg M \quad \frac{L \wedge M}{R}}{R}</math></p>                  | <p>16. <math>\frac{(M \wedge L) \rightarrow (P \vee R) \quad (L \wedge M) \wedge \neg R}{P}</math></p>  |
| <p>4. <math>\frac{\neg(\neg J \vee K) \rightarrow \neg(L \wedge M) \quad \neg(\neg J \vee K) \wedge M}{\neg(L \leftrightarrow M)}</math></p> | <p>17. <math>\frac{L \rightarrow [M \rightarrow (\neg N \vee I)] \quad (M \wedge \neg I) \wedge L}{\neg N}</math></p>   |
| <p>5. <math>\frac{R \rightarrow (\neg C \rightarrow D) \quad \frac{\neg C \wedge R}{D \wedge R}}{D \wedge R}</math></p>                      | <p>18. <math>\frac{[(B \vee M) \wedge R] \rightarrow T \quad \frac{M \wedge R}{T}}{T}</math></p>  |
| <p>6. <math>\frac{J \wedge K \quad K \wedge (J \vee P)}{K \wedge (J \vee P)}</math></p>  | <p>19. <math>\frac{K \wedge \neg L \quad [\neg(\neg G \wedge \neg H) \vee R] \wedge F \quad \frac{F \rightarrow \neg R}{Q \vee \neg(\neg G \wedge \neg H)}}{Q \vee \neg(\neg G \wedge \neg H)}</math></p> |
| <p>7. <math>\frac{F \wedge \neg I \quad \frac{H \vee I}{H \wedge F}}{H \wedge F}</math></p>  | <p>20. <math>\frac{(A \wedge \neg C) \wedge (E \wedge F) \quad (A \wedge F) \wedge E}{(A \wedge F) \wedge E}</math></p>   |
| <p>8. <math>\frac{[M \vee (C \rightarrow T)] \wedge P \quad (P \rightarrow \neg M) \wedge C}{T}</math></p>                                   | <p>21. <math>\frac{C \wedge (\neg R \wedge S) \quad M \wedge (\neg R \rightarrow P)}{P \wedge C}</math></p>   |
| <p>9. <math>\frac{\neg(\neg M \vee \neg B) \wedge P}{\neg(\neg M \vee \neg B) \vee \neg P}</math></p>  | <p>22. <math>\frac{C \rightarrow [(C \wedge D) \rightarrow (A \rightarrow B)] \quad D \rightarrow C \quad \frac{D \wedge B}{A \rightarrow B}}{A \rightarrow B}</math></p>                                 |
| <p>10. <math>\frac{E \quad (R \vee E) \wedge (E \vee \neg(N \vee M))}{(R \vee E) \wedge (E \vee \neg(N \vee M))}</math></p>                  | <p>23. <math>\frac{E \rightarrow (F \wedge \neg G) \quad J \vee \neg H \quad \frac{E \wedge F}{\neg G}}{\neg G}</math></p>  |
| <p>11. <math>\frac{(O \vee Q) \rightarrow R \quad (F \wedge Q) \wedge C}{F \wedge R}</math></p>  | <p>24. <math>\frac{(L \wedge B) \wedge (Q \vee F) \quad \neg Q \wedge [(F \wedge L) \rightarrow R]}{R}</math></p>   |
| <p>12. <math>\frac{M \wedge (O \vee N) \quad (O \vee N) \wedge (M \vee O)}{(O \vee N) \wedge (M \vee O)}</math></p>                          | <p>25. <math>\frac{\neg L \vee M \quad \frac{M \rightarrow N}{L \rightarrow N}}{L \rightarrow N}</math></p>   |
| <p>13. <math>\frac{A \vee D \quad \neg S \rightarrow C \quad A \rightarrow \neg S \quad \frac{\neg D}{C}}{C}</math></p>                      |   |

$$26. \frac{A \rightarrow B}{B \rightarrow \neg C}$$

$$\frac{\neg D \rightarrow E}{\neg C \rightarrow \neg D}$$

$$\frac{\neg E \rightarrow \neg A}{\neg E \rightarrow \neg A}$$

$$27. \frac{\neg(H \vee U)}{\neg(H \vee \neg C)}$$

$$\frac{\neg(C \wedge E)}{\neg(H \vee E)}$$

$$28. \frac{F \vee F}{F \wedge F}$$

$$29. \frac{O}{M \rightarrow (Q \rightarrow (L \rightarrow O))}$$

$$30. \frac{(P \rightarrow Q) \rightarrow (R \rightarrow S)}{R \wedge \neg S}$$

$$\frac{\neg Q}{\neg Q}$$

$$31. \frac{R \rightarrow (U \wedge P)}{E \vee R}$$

$$\frac{E \rightarrow F}{F \leftrightarrow U}$$

$$\frac{U}{U}$$

$$32. \frac{C \wedge \neg F}{(P \rightarrow H) \leftrightarrow I}$$

$$\frac{H \leftrightarrow \neg F}{I}$$

$$33. \frac{A \wedge (B \vee C)}{(A \wedge B) \vee (A \wedge C)}$$

$$34. \frac{(L \rightarrow \neg M) \rightarrow P}{Q \wedge \neg P}$$

$$\frac{M \wedge O}{M \wedge O}$$

$$35. \frac{\neg(Q \wedge C)}{D \rightarrow C}$$

$$\frac{\neg Q \rightarrow \neg D}{\neg D}$$

$$36. \frac{E \rightarrow (F \rightarrow G)}{E}$$

$$\frac{\neg F \rightarrow \neg E}{G}$$

$$37. \frac{\neg(M \vee N)}{\neg(M \leftrightarrow \neg N)}$$

$$38. \frac{(D \rightarrow E) \rightarrow D}{D}$$

$$39. \frac{B \rightarrow C}{\neg C \rightarrow \neg(B \wedge G)}$$

$$40. \frac{R \rightarrow S}{\neg R \rightarrow \neg S}$$

$$\frac{R \leftrightarrow S}{R \leftrightarrow S}$$

$$41. \frac{(Q \rightarrow S) \wedge \neg N}{(\neg Q \rightarrow O) \vee N}$$

$$\frac{(\neg S \rightarrow O) \rightarrow L}{L}$$

$$42. \frac{B \vee \neg C}{\neg B \vee \neg C}$$

$$\frac{\neg C}{\neg C}$$

$$43. \frac{H \vee (F \vee D)}{D \vee (F \vee H)}$$

$$44. \frac{(C \vee E) \vee F}{\neg C \rightarrow (\neg E \rightarrow F)}$$

$$45. \frac{J \wedge (\neg K \rightarrow L)}{(J \wedge K) \vee (J \wedge L)}$$

$$46. \frac{R \leftrightarrow S}{(S \vee T) \wedge \neg T}$$

$$\frac{R}{R}$$

$$47. \frac{G \leftrightarrow \neg H}{\neg H \leftrightarrow I}$$

$$\frac{I \leftrightarrow \neg E}{\neg E \rightarrow G}$$

$$48. \frac{[M \vee (O \leftrightarrow T)] \wedge N}{N \leftrightarrow \neg M}$$

$$\frac{B \rightarrow (O \leftrightarrow T)}{B \rightarrow (O \leftrightarrow T)}$$

$$49. \frac{\neg S \leftrightarrow C}{(F \vee S) \wedge (G \wedge M)}$$

$$\frac{G \rightarrow \neg S}{M \wedge C}$$

$$50. \frac{(A \wedge B) \leftrightarrow (P \vee D)}{(B \wedge A) \wedge \neg D}$$

$$\frac{P}{P}$$