Introductory Practice Natural Deduction Proofs
PHI 154 (Eliot) Fall 2023 (version 2023-11-06)

Proofs without subproofs

Proving these arguments does not require any subproofs. That is, no additional assumptions are required. The basic TFL deduction rules you may need are just ∧I, ∧E, →E, ↔E and ∨I. R is also available, but you won't need it for these.

1. A
   B
   $\overline{A \wedge B}$

2. $C \wedge D$
   $\overline{C}$

3. $B \wedge A$
   $\overline{A \wedge B}$

4. D
   E
   F
   $\overline{(E \wedge D) \wedge F}$

5. $C \wedge (D \wedge E)$
   $\overline{E \wedge C}$

6. Q
   Y
   V
   $\overline{(Y \wedge Q) \wedge (Y \wedge V)}$

7. $(H \wedge J) \wedge (I \wedge K)$
   $\overline{K \wedge J}$

8. $(O \wedge M) \wedge N$
   $\overline{N \wedge M}$

9. $L \rightarrow N$
   L
   $\overline{N}$

10. $O \wedge R$
    $R \rightarrow S$
    $\overline{S \wedge O}$

11. $R \rightarrow (B \wedge A)$
    R
    $\overline{B}$

12. $T \wedge (D \rightarrow E)$
    $R \wedge D$
    $\overline{E}$

13. $(W \rightarrow E) \wedge (C \wedge R)$
    $W \wedge G$
    $\overline{E}$

14. $(A \rightarrow C) \wedge (B \rightarrow D)$
    $A \wedge B$
    $\overline{C \wedge D}$

15. $H \wedge A$
    $H \rightarrow L$
    $A \rightarrow M$
    $\overline{M \wedge L}$

16. $M \wedge (C \rightarrow D)$
    $(C \rightarrow N) \wedge C$
    $\overline{N \wedge M}$

17. C
    $\overline{C \vee M}$

18. $D \wedge A$
    $\overline{A \wedge (D \vee B)}$

19. $P \wedge M$
    $(M \vee Z) \rightarrow X$
    $\overline{X}$

20. $(K \wedge C) \wedge (H \rightarrow U)$
    $(C \rightarrow M) \wedge L$
    $\overline{(K \wedge L) \wedge (M \vee U)}$

21. $R \leftrightarrow F$
    $R \wedge K$
    $\overline{F}$

22. $R \leftrightarrow F$
    $F \wedge G$
    $\overline{R}$

23. $L \leftrightarrow H$
    $L \leftrightarrow M$
    H
    $\overline{M}$

24. $V \leftrightarrow X$
    $T \leftrightarrow Y$
    $Y \wedge X$
    $\overline{T \wedge V}$

Proofs with some subproofs

Proving these arguments may require subproofs. That is, additional assumptions may be required. Besides the rules assumed in the previous section, the additional natural deduction rules assumed are $\rightarrow$I, $\leftrightarrow$I, $\vee$E, and R. The negation rules are not assumed here.

25. $\dfrac{L}{D \rightarrow L}$

26. $\dfrac{A \wedge B}{D \rightarrow A}$

27. $\dfrac{C}{(M \wedge K) \rightarrow C}$

28. $\dfrac{F \wedge G}{G \rightarrow F}$

29. $\dfrac{A \wedge B}{N \rightarrow (B \wedge A)}$

30. $\dfrac{M \rightarrow (P \wedge D)}{M \rightarrow D}$

31. $\dfrac{(Q \rightarrow Z) \wedge Q}{F \rightarrow (F \wedge Z)}$

32. $\dfrac{V}{N \rightarrow (V \wedge N)}$

33. $\dfrac{C}{(H \rightarrow C) \wedge (I \rightarrow C)}$

34. $\dfrac{G}{(L \wedge M) \rightarrow (G \vee R)}$

35. $\dfrac{T}{(T \rightarrow S) \rightarrow (S \wedge T)}$

36. $\dfrac{H \wedge R}{L \rightarrow (R \wedge H)}$

37. $\dfrac{Z}{X \rightarrow (Y \rightarrow Z)}$

38. $\dfrac{T \leftrightarrow B}{B \rightarrow T}$

39. $\dfrac{C \wedge D}{C \leftrightarrow D}$

40. $\dfrac{E \wedge F}{F \leftrightarrow E}$

41. $\dfrac{L \wedge (L \rightarrow M)}{M \leftrightarrow L}$

42. $\dfrac{(O \rightarrow M) \wedge (M \rightarrow O)}{O \leftrightarrow M}$

43. $\dfrac{(A \rightarrow C) \wedge (C \rightarrow A)}{C \leftrightarrow A}$

44. $\dfrac{R \leftrightarrow S}{S \leftrightarrow R}$

45. $\dfrac{\begin{array}{l} S \leftrightarrow E \\ L \leftrightarrow F \end{array}}{(F \wedge E) \rightarrow (S \wedge L)}$

46. $\dfrac{\begin{array}{l} A \vee B \\ A \rightarrow C \\ B \rightarrow C \end{array}}{C}$

47. $\dfrac{M \vee D}{D \vee M}$

48. $\dfrac{\begin{array}{l} F \vee G \\ G \leftrightarrow H \\ L \leftrightarrow F \end{array}}{H \vee L}$

49. $\dfrac{\begin{array}{l} L \leftrightarrow M \\ M \end{array}}{D \rightarrow (L \wedge D)}$

50. $\dfrac{\begin{array}{l} M \vee N \\ M \rightarrow O \end{array}}{O \vee N}$