

FOL Natural Deduction Exercise
 PHI 154 (Eliot) Fall 2021

Write natural deduction proofs for the following valid arguments using the rules of FOL. The natural deduction rules of TFL are presupposed. From FOL, only rules $\forall E$ and $\exists I$ are required for this exercise. (Questions 7–12 are adapted from arguments written by Paul Teller, who licenses them for this use.)

$$1. \frac{T(e)}{\exists x T(x)}$$

$$2. \frac{C(g)}{\exists x C(x) \wedge \exists y C(y)}$$

$$3. \frac{K(h) \wedge M(h)}{\exists z (K(z) \wedge M(z))}$$

$$4. \frac{\forall y (\neg K(y) \leftrightarrow L(y)) \quad L(m)}{\neg K(m)}$$

$$5. \frac{\forall y \neg M(y)}{\exists y \neg M(y)}$$

$$6. \frac{\forall x (C(x) \vee D(x)) \quad \forall y \neg D(y)}{\exists z C(z)}$$

$$7. \frac{\exists x R(x, a) \rightarrow \forall x R(a, x) \quad R(e, a)}{\exists x R(a, x)}$$

$$8. \frac{L(a, e) \vee L(e, a) \quad \exists x L(a, x) \rightarrow A \quad \exists x L(x, a) \rightarrow A}{A}$$

$$9. \frac{\exists x J(x) \rightarrow Q \quad \forall x J(x)}{Q}$$

$$10. \frac{\forall x (M(a, x) \vee M(e, x)) \quad \neg \exists x M(a, x) \vee B(g) \quad \neg \exists x M(e, x) \vee B(g)}{\exists x B(x)}$$

$$11. \frac{\forall x (K(x, x) \leftrightarrow P(x)) \quad \forall x [K(j, x) \wedge (P(x) \rightarrow S(x))]}{\exists x S(x)}$$

$$12. \frac{\forall x (\neg O(x, x) \vee I(x)) \quad \forall x (I(x) \rightarrow R(x, m))}{\forall x O(x, x) \rightarrow \exists x R(x, m)}$$