Eight Ways of Looking at Validity

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While in ordinary English "valid" means "acceptable" or "legitimate," in logic "logically valid" means something more specific. It is a central concept of logic. It is important to understand it before we go further. This handout explains the concept in different ways. It is repetitive; that is sort of the point.

In this handout and in the course we will often just say "valid" and "invalid" when we mean "logically valid" and "logically invalid."

Logical validity is a property of some arguments.[1] Your learning how to recognize whether arguments are logically valid is the main goal of this course.

Logically valid arguments do something valuable: their premises *logically entail* their conclusions. We call such arguments "logically valid." Here are ways of thinking about what "logical validity" means:

[1] Note that logical validity is *not* a property of any sentences. Sentences may be true or false, but may be neither valid nor invalid. Similarly, we never describe arguments as true or false. Arguments are sets of sentences which may have different truth-values. We can say of sets that they are consistent or inconsistent, and we can describe arguments as valid, invalid, sound, and unsound.

1. A typical, compact definition of "logical validity" is the *The Logic Book's*:

   > An argument is *logically valid* if and only if it is not possible for all the premises to be true and the conclusion to be false.[2] Relatedly, an argument is *logically invalid* if and only if it is not logically valid.

   [2] *The Logic Book* 6th ed., pages 5, 14.

2. The *forall x* book's definition of logical defines (logical) validity in three stages:

   (a) "An argument is VALID if and only if the conclusion is a consequence of the premises."[3]

   [3] *forall x*, page 9

   (b) This definition of "validity" requires that we define "consequence":

   > A sentence A is a CONSEQUENCE of sentences $B_1, \ldots, B_n$[4] if and only if there is no case where $B_1, \ldots, B_n$ are all true and A is not true.

   [4] This notation refers to an itemized set of sentences with n members. So, think of an argument like:
   B_1
   B_2
   $\frac{B_3}{A.}$

   (c) This definition of "consequence" in turn depends on a definition of "case"! Roughly, a CASE is a "hypothetical scenario" or a "conceivable scenario." But that is not an exhaustive definition.[5] A better definition of "case" is: an assignment of specific truth-values—of either True or False—to every sentence in the argument.

   [5] for reasons the *forall x* book explains in sections 2.2 and 2.3

3. These definitions say that the sentences that make up a *valid* argument cannot, considered together, have a certain combination of truth values: the premises can't all be true while the conclusion is false. But wait, you might think, *any* argument with false premises and a false conclusion avoids that combination! For example:

   Joe Biden is an acrobat. (false premise)
   The city of Tokyo is in Ohio. (false conclusion)

   So, is that argument valid? No. Here's where the other part of the definition becomes important: the definition says that in valid arguments the combination of true premises and false conclusion is *impossible*, not just that it didn't actually happen.

Well, how can it be impossible? The answer is logical structure. The above argument has premises that are not true while the conclusion is false, sure. It *avoids* that combination, but its structure does not *prevent* it. It lacks a structure that would prevent the premises from being true while the conclusion is false (or equivalently, prevent the conclusion from being false when the premises are true). Accordingly, it lacks a structure that would make it logically valid.

As we're using the word "possibility" here, it *is possible* for both of the sentences above to be true or false. Joe Biden could have become an acrobat. Moreover, the kind of possibility we're discussing is not limited to what could actually have happened. We're interested in everything that can be the case without contradiction.[6] So, we're understanding it to be possible, in this sense, for Joe Biden to have been thirteen feet tall or made of steel. Most kinds of actually true sentences could, in this sense, have been false,[7] and vice versa.

In this inclusive sense of possibility, it is possible for that premise to be true and that conclusion to be false.[8] You might wonder: So, if we're going to be that flexible about possible truth-values, how could it ever be *impossible* for some premises to be true while the conclusion is false? How could a true premise ever prevent a conclusion from being false? Look at this example:

Tokyo is a city in Japan.

Tokyo is a city.

If that argument's premise is true, it is impossible for its conclusion to be false *because of its form*, or equivalently because of its logical structure. We can expose this logical structure by substituting in variables for the words. Now we see that no matter what terms we plug in below for α and β, the argument is valid:[9]

α is a city in β.

α is a city.

4. "A logically valid argument is truth-preserving. If the premises are true, then the conclusion must also be true. […] Note that to determine validity, we do not need to know whether the premises or conclusion are in fact true. All we need to know is the logical relation between the premises and the conclusion."[10]

5. "We are not concerned with whether the premises or conclusion [of a given argument] *are* in fact true; all that is required for [validity] is that *if* the premises were true, the conclusion would have to be true. Another way of stating the same criterion is this: an argument is [valid] if and only if every conceivable circumstance that would make the premises true would also make the conclusion true."[11]

[6] We can call this kind of possibility "logical possibility" as opposed to what's physically possible or historically possible, or possible in other ways.

[7] Which true sentences could have been false? Sentences that are "logically indeterminate," and thus not logical truths (not, that is, what we call "tautologies").

[8] And that is precisely why the argument is invalid.

[9] So yes, this silly argument, for instance, is valid:

Joe Biden is a city in Antarctica.

Joe Biden is a city.

But also, by the same token this less silly argument is valid, too:

Buffalo is a city in New York.

Buffalo is a city.

That is a valid argument *and*, because its one premise is true, it has all true premises. We call such arguments "sound." And because it is a sound argument, it necessarily also has a true conclusion.

If you haven't seen these symbols before, 'α' is a lowercase Greek 'alpha' and 'β' is a lowercase Greek 'beta'; here they're just variables where we can plug in any names or nouns.

[10] *The Logic Book*, page 5.

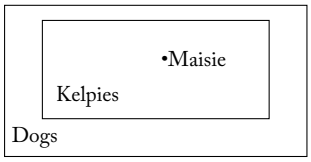
[11] Benson Mates, 1972. *Elementary Logic*, page 5.

6. "[An example]:

   All kelpies are dogs.
   Maisie is a dog.
   Therefore, Maisie is a kelpie.

   Can we imagine a situation in which the premises are both true but the conclusion is false? Yes: suppose that (as in actual fact) all kelpies are dogs (so the first premise is true) and suppose that Maisie is a beagle (and hence a dog—so the second premise is true); in this case the conclusion is false. Hence [the argument is invalid]. Now consider [another] example:

   All kelpies are dogs.
   Maisie is a kelpie.
   Therefore, Maisie is a dog.



   Can we imagine a situation in which the premises are both true but the conclusion is false? No. Supposing the first premise to be true means supposing that (to represent the situation visually) a line drawn around all kelpies would never cross outside a line drawn around all dogs.

   Supposing the second premise to be true means supposing that Maisie is inside the line drawn around the dogs—that is, it is impossible for the conclusion to be false. So, [this argument is valid]."[12]

[12] Nicholas J. J. Smith, 2012. *Logic: The Laws of Truth*, pages 14–15.

7. "Logic is about many things, but most centrally it is about *logical consequence*. The statement 'someone is male' is a logical consequence of the statement 'Grant is male.' If Grant is male, then it *logically follows* that someone is male. Put another way: the statement 'Grant is male' *logically implies* the statement 'someone is male.' Likewise, the statement 'Grant is male' is a logical consequence of the statements 'It's not the case that Leisel is male' and 'Either Leisel is male or Grant is male' (taken together). The first statement *follows from* the latter two statements; they logically imply it. Put another way: the argument whose premises are the latter two statements, and whose conclusion is the former statement is a *logically correct* one. (The word 'valid' is sometimes used for logically correct arguments.)"

   "So far we've just given synonyms. The following slogan advances us a bit further: *logical consequence is truth-preservation by virtue of form*. To say that an argument 'preserves truth' is to say that *if* its premises are true, *then* its conclusion is also true. 'By virtue of form' requires that the truth preservation be due solely to the form, not the content, of the argument.[13] For example, if you drop an apple, it will fall; so in a sense the inference from 'you drop the apple' to 'the apple falls' is truth-preserving. But the former does not logically imply the latter because the premise is about dropping and the conclusion is about falling. By contrast, the fact that

[13] Sider clearly expresses in this paragraph the kind of possibility and impossibility that is relevant to logical validity.

the argument from 'Grant is male' to 'Someone is male' has nothing to do with the fact that its content concerns Grant and maleness; *any* argument of the form 'α is F; therefore someone is F' preserves truth."[14]

[14] Ted Sider, 2010. *Logic for Philosophy*, pages 1–2.

8. On first encountering logical validity, you might be concerned that we're describing as valid some arguments that are clearly, in a more general way, *bad arguments*. Some examples are: arguments with premises or conclusions that can't actually be true or don't make sense in the real world,[15] arguments with logical falsehoods in their premises, arguments with contradictory premises, arguments with logical truths in their conclusions. Yes, validity is not the only thing we look for in arguments as we use them in the real world. We also value things like truth, consistency, clarity, and understandability. Validity is just what we value *logically*.

[15] The argument about Biden and Antarctica in footnote 9 is an example.

You might at this point be wondering something like this: now I know the definition of logical validity, but I'm having a hard time thinking of my own examples because I don't know how to tell which arguments are the ones where:

- truth is preserved by virtue of form or structure,
- premises logically entail conclusions,
- the arguments succeed at being logically valid.

← All three of these phrases mean the same thing.

Hang tight, folks; that's what this course is about.