Exercises in Existential and Universal Quantification
PHI 154 (Eliot, Fall 2022)
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domain = people (including characters who are maybe not exactly human beings!)
$C(x) = x$ counts sheep
$D(x) = x$ has a ducky
$G(x) = x$ is grumpy
$H(x) = x$ is happy
$K(x) = x$ is a kid
$O(x) = x$ wears orange
$F(x, y) = x$ is friends with y
$L(x, y, z) = x$ likes y better than z
b = Bert
c = The Count
e = Ernie
o = Oscar

Using the symbolization key above, translate these from First-order Logic into English. Think about them literally first (if that's useful), and then think whether there is a more natural way to express them in English. You can use both the universal and existential quantifiers.

1. $\forall x L(x, e, b)$
2. $\forall x \neg D(x)$
3. $\neg \forall x G(x)$
4. $\neg \forall y \neg H(y)$
5. $\forall z[(D(z) \wedge C(z)) \wedge F(z, e)]$
6. $\forall y(O(y) \to \neg G(y))$
7. $\forall y[D(y) \to (H(e) \wedge H(y))]$
8. $\forall z[(H(z) \wedge K(z)) \to O(z)]$
9. $\exists x(H(x) \wedge K(x)) \leftrightarrow \forall y[K(y) \to (O(y) \vee D(y))]$
10. $\forall x \neg D(x) \to \exists y(K(y) \wedge \neg H(y))$
11. $(H(c) \wedge O(c)) \leftrightarrow \forall z(C(z) \to H(z))$
12. $\exists y L(y, e, o) \to \forall z(K(z) \to \neg G(z))$
13. $\forall x L(x, e, b) \to [\neg \forall z G(z) \wedge G(b)]$
14. $L(b, e, b) \to [G(b) \wedge \neg \forall x(K(x) \to L(x, e, b))]$
15. $[F(o, o) \to \exists x F(x, x)] \wedge \neg \forall z L(z, o, z)$
16. $\forall x(L(x, x, o) \to \neg F(o, x))$
17. $\forall y L(o, o, y) \wedge \forall z L(z, c, o)$
18. $\forall x[L(b, e, x) \to (L(x, e, b) \wedge \neg(G(x) \vee G(b)))]$
19. $\forall z\{[(H(z) \wedge K(z)) \to \exists y[(H(y) \wedge K(y)) \wedge F(z, y)]\}$
20. $\forall x[(F(x, e) \vee F(x, b)) \to (F(c, x) \wedge L(c, c, x))]$

Translate from English into Predicate Logic. You can use both the universal and existential quantifiers.

1. If Ernie doesn't have a ducky, someone's not happy.
2. Anyone who counts sheep is friends with The Count.
3. Every kid who wears orange has a ducky or counts sheep.
4. Ernie is friends with anyone who wears orange.
5. Only if Ernie is wearing orange and has a ducky, everyone is happy.
6. If someone is grumpy, not everyone is happy.
7. Were everyone to count sheep, nobody would be grumpy.
8. If everyone wears orange, not everyone is happy, but Ernie is.
9. If anyone is friends with themselves, Ernie is friends with them.
10. If a kid has a ducky, everyone is friends with that kid.
11. Kids like Ernie more than Oscar.
12. Happy people have duckies or count sheep.
13. No grumpy kids have a ducky.
14. No happy kids don't wear orange.
15. Kids count sheep only if they like The Count better than Bert.
16. Anyone who doesn't have a ducky isn't friends with Ernie.
17. Nobody counts sheep unless The Count does.
18. Nobody who doesn't count sheep is friends with The Count.
19. Anyone who's friends with The Count likes The Count better than they like themselves.
20. Kids who have duckies are friends with Ernie, and kids who count sheep are friends with The Count, but those kids with duckies aren't friends with those kids who count sheep.*

* This one requires doing something you haven't seen done yet, syntactically, but which is allowed by the rules of PL as we've set them up. Something like it is in #19 on the first page. Can you see what that is? Try it and see what you get.