What Material Conditionals Mean

PHI 154 (Eliot) Fall 2023

version 2.3 2023-09-27

The material conditional ($\rightarrow$) expresses the basic if-then relationship in symbolic logic. However, its meaning beyond "if" and "then" confuses many students at first, and is worth spending time to understand. This handout presents its semantics, especially its relationship to the useful concept of necessary and sufficient conditions.

The familiar interpretation of the material conditional "$P \rightarrow Q$" is "If P, then Q." If you've thought about the conditional at all, you know that!

However, the material conditional is not the only kind of if-then. That is, it is not the only kind of conditional. Other kinds of conditionals express *causation* ("If P happens, it will cause Q to happen") or *temporal sequences* ("If P happens, Q will follow"). The material conditional does not. It asserts a truth-functional relationship[1] between statements about states of affairs without being committed to their being connected to one another in any further way. The relationship it asserts is completely captured by its characteristic truth table (Table 1).

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 1: The characteristic (defining) truth table for the material conditional

[1] It being truth-functional means that its truth value is a function *only* of the truth values of its component sentences.

To think about the meaning of this truth-table all at once: $P \rightarrow Q$ means that[2] if P is true,[3] Q must be also,[4] while if P is false,[5] Q can go either way. The conditional is 'noncommittal' about Q in cases where P is false. $P \rightarrow Q$ also *does not* assert that P *is* true (or that Q is). It asserts that *if* P is true, Q is also true.

The sentence on the left side of a conditional is called its "antecedent." The sentence on the right side is called its "consequent." For simplicity, I'm going to keep referring to them here as P and Q.

[2] Here I mean that the whole compound sentence "$P \rightarrow Q$" means this. From here on, I won't be careful about putting mentioned sentences in quotation marks.

[3] truth table rows 1 and 2

[4] row 1

[5] rows 3 and 4

Here's an example of a true conditional sentence. For brevity, we'll also call such a sentence a "conditional." Let's associate B with "You're in Brooklyn," and N with "You're in New York":

$B \rightarrow N$: If you're in Brooklyn, you're in New York.

Looking at the cases where it is true in Table 1 from top to bottom, $B \rightarrow N$ tells you that if you're in Brooklyn then you're in New York, and if you're not in Brooklyn, you may or may not be in New York. The one scenario this rules out is that you are in Brooklyn but not in New York. If the conditional is true, that scenario is not possible.[6]

[6] An occasional source of confusion is thinking of these sentence-definitions as having the kind of ambiguity English sentences have. What if you're in Brooklyn, *Iowa*, you might ask? Then it would be possible to be in Brooklyn but not in New York. The answer is that when we translate sentences into symbolized form, we treat them as unambiguous. It's crucial that we agree on exactly what they mean.

Here's an example of a false conditional. Let's now associate M with "You're in Minnesota":

$B \rightarrow M$: If you're in Brooklyn, you're in Minnesota.

If we know that the conditional is false, we know that it is possible to be in Brooklyn and not be in Minnesota (row 2 on Table 1). The conditional is false just when that case can occur.

Sufficient conditions

Another way of expressing the if-then inference the material conditional licenses is to say that the truth of its antecedent (P) is *sufficient* for the truth of its consequent (Q). Or, the truth of P offers sufficient grounds for inferring the truth of Q. Or, in epistemic[7] terms, the truth of P is sufficient evidence for concluding Q. If you know P, you can conclude Q. We say P is a sufficient condition for Q because in these ways P is *enough* for Q. You don't need anything more. If you find that you're in Brooklyn, that's sufficient grounds for inferring that you're in New York.

[7] i.e., having to do with knowledge and its justification

Besides "If P then Q" some of the ways sufficient conditions are expressed in English are: P is grounds for Q; P entails Q; given P, Q; provided that P, Q; Q if P; Q, given P; Q follows from P; Q is entailed by P.[8]

[8] Two related sentences *not* on this list are "Because P, Q" and "Since P, Q." I think those sentences assert, in addition to $P \to Q$, that P is true. Moreover, they say that P being true is the reason Q is true. The material conditional does not say that. It does not say there there is any relationship between P and Q that goes beyond how their truth values are related. By the same token, it also doesn't say that they have no such relationship!

Note that $P \to Q$ does *not* say that Q is sufficient for P. In $B \to N$, being in New York is not sufficient for being in Brooklyn. You might be in Poughkeepsie or Schenectady or Montauk. Knowing that Q is true leaves open whether P is true (row 1) or false (row 3). If you're in New York, you might be in Brooklyn, but you also might not be. So again, $P \to Q$ does not say that Q is sufficient for P.

Necessary conditions

A "necessary condition for X" is a condition that needs to be true for X to be true. Some examples: being a dog is a necessary condition for being a good dog; being human is a necessary condition for being a woman; being over 16 is a necessary condition for having a driver's license in New York; understanding what a line is is a necessary condition for understanding what a triangle is.

Being a necessary condition is *not* the same as being a sufficient condition, and the two are often not true at the same time. Being a dog is a necessary condition for being a good dog, but it is not sufficient. Being a good dog is sufficient for being a dog, but it is not necessary.

Here we arrive at the main common point of confusion. Helping some students get clear on this idea was my motivation for writing this.

Here is the point I have been leading up to: the material conditional expresses *both* sufficient and necessary conditions, but in *opposite directions*. As above, $P \to Q$ says that P is sufficient for Q, and does not say that Q is sufficient for P. Equivalently, the antecedent is sufficient for the consequent, but the consequent is not sufficient for the antecedent. However, it also says that Q is necessary for P. That is, in addition to saying "If P then Q" it says "P only if Q." The consequent is necessary for the antecedent, though the antecedent is not sufficient for the consequent.

P	Q	$P \to Q$
T	$\boxed{T}$	T
T	F	F
F	T	T
F	F	T

Table 2: Where the conditional and the antecedent are true, the consequent necessarily is, too. It is a necessary condition for the antecedent.

Why? Look again at the truth table. If $P \to Q$ is true (rows 1, 3, and 4), P is true on those rows only when Q is (row 1). So Q needs to be true for P to be true, and Q is a necessary condition for P. That is, P only if Q.

Some of the ways we express that Q is a necessary condition for P in English are: P only if Q; Q is necessary for P; Q is essential for P; you can't have/have evidence for/be P without having/having evidence for/being Q.[9]

[9] This is the basis for the argument form called *modus tollens* or denying the consequent, which we will encounter shortly.

Necessary and sufficient conditions

We're now in a position to put necessary and sufficient conditions together. Determining necessary and sufficient conditions for things is a very common task in philosophy, as well as law, science, and other disciplines. When we have both necessary and sufficient conditions for X, we have a definition for X. We know what X is equivalent to.

For example, the necessary and sufficient conditions for being beer are: being a malted beverage made from water, malt, hops, and yeast. Being a lager[10] is a sufficient but not necessary condition for being a beer; there are other kinds of beer like ale, stout, and porter, so a beer doesn't need to be a lager. Being a malted beverage is a necessary but not sufficient condition for being a beer. Wine coolers are also malted beverages, so while beer needs to be a malted beverage, being a malted beverage isn't enough to make something a beer.[11] When we know necessary and sufficient conditions for something, we know exactly what it is.

[10] a lighter-colored style of beer including Budweiser and MGD

[11] Other features of some beers, like added fruit or chocolate, are neither necessary nor sufficient for being beer.

A philosophical example: Plato proposes that we have knowledge exactly whenever we have justified true beliefs.[12] Another way of saying this is that we have knowledge if and only if we have justified true beliefs. Having a true belief is necessary but not on its own sufficient for having knowledge. The belief must also be justified. Knowing that Earth is round is sufficient for having knowledge, but not necessary (and accordingly no specific facts appear in the definition of knowledge). Plato would say that having a belief, the belief being true, and the belief being justified are jointly sufficient conditions for knowledge. They are also all required for knowledge, and so are each necessary conditions.[13]

[12] In Plato's *Theaetetus*, a dialogue which is all about knowledge, Socrates says at *210b*: "So, it seems, the answer to the question 'What is knowledge?' will be 'Correct judgement accompanied by … an account.'" The common interpretation of 'account' is 'justification.' The Hackett version of *Theaetetus* (1990) translated by M.J. Levett contains an outstanding, intellectually exciting commentary by Myles Burnyeat.

[13] It's common for definitions to involve several jointly sufficient but individually necessary conditions, like this one does.

To express necessary and sufficient conditions for some one thing in terms of material conditionals, we need two conditionals. $P \to Q$ says that P is a sufficient condition for Q. $Q \to P$ says that P is a necessary condition for Q. So P is necessary *and* sufficient for Q when $(P \to Q) \wedge (Q \to P)$. Since it is so often useful to express necessary and sufficient conditions, there is a symbol that expresses them. It's the "biconditional" ($\leftrightarrow$). Adding the "bi" makes it mean "two conditionals." $P \leftrightarrow Q$ means "P if and only if Q."

Other ways to write the biconditional are 'IFF' (short for 'if and only if') and the symbol "$\equiv$" which is often used in texts which use "$\supset$" instead of "$\to$" for the material conditional.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Table 3: The characteristic truth table for the material *biconditional*.

Phrases that express biconditionals in English include: P if and only if Q; P just in case Q; P exactly when Q; P if but only if Q; P is the definition of Q; P is equivalent to Q.

So, quick: in "If you're in Brooklyn, you're in New York," what's sufficient for what? What's necessary for what?

And you are now in a position to understand exactly why the two sides of this sentence each express "P if and only if Q":

$$(P \leftrightarrow Q) \leftrightarrow [(P \to Q) \wedge (Q \to P)]$$